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INSTABILITY CRITERIA IN THE STUDY OF MINIMIZING ROLLING AND PITCHING

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INSTABILITY CRITERIA IN THE STUDY OF MINIMIZING ROLLING AND PITCHING

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ABSTRACT

The 60°-sweep delta-wing Griffon aircraft was tested by analog computer to increase speed to Mach 2.5. Pitch coupling and stabilized values of roll rate were studied. Five nonlinear algebraic equations were used for slip, incidence, and angular rates of roll, pitch and yaw. Some use of digital computer was made. Stable and unstable movement are considered. author

The present study was undertaken in 1960, with reference to the Griffon aircraft. This 60°-sweep delta wing, propelled by a turbostatoreactor assembly, had already exceeded Mach 2, and it was then a question of extending its flight capability to Mach 2.5 or more by certain structural modifications. A classic study of the problem of pitch coupling was therefore conducted, using an analog computer. /1*

In analyzing these results it seemed to be of interest to attempt an analytic study to explain certain deviations. The idea was to examine the stabilized values of roll rate and to see the evolution of this parameter.

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*Numbers given in margin indicate pagination in original foreign text.
[1]Nord-Aviation

We thought that in certain areas there would be only infinite roots, which would indicate instability.

The main hypotheses are fairly standard, but we recall here:

-- The equations are written in the system of principal axes of inertia.

-- Weight is disregarded.

-- Velocity and altitude are assumed to be constant; the equation of /2 longitudinal force is thus eliminated.

-- Incidence i and slip j are small angles.

-- The aerodynamic derivatives are constant: a variation of C_{l_j} and C_{n_j} with i is often considered, but it was found to be impossible in this case.

-- There is an aileron deflection and an elevator deflection.

We thus have five equations, i.e., lateral force, normal force and three equations of moment of roll, pitch and yaw. Since we are seeking stabilized values, we cancel the derivatives of the various variables. We thus have five nonlinear algebraic equations for the five variables: slip, incidence, angular rates of roll p, pitch q and yaw r.

They are written analytically as follows:

$$pi + a_2 j + a_3 p + a_5 r = 0 \qquad (1)$$

$$pj + b_1 i + b_4 q = B\beta \qquad (2)$$

$$qr + c_2 j + c_3 p + c_5 r = C\alpha \qquad (3)$$

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$$rp + d_1 i + d_4 q = D\beta \qquad (4)$$

$$pq + e_2 j + e_3 p + e_5 r = E\alpha \qquad (5)$$

The successive eliminations of i, q, j and r bring a conclusion with no difficulty other than the analytic complexity of writing an equation of the

11-th degree in p. For various cases of calculation already worked out on the analog computer, we resolved this question on a digital computer.

We have always found three real roots, one of which varies linearly with the aileron deflection α, and which is in excellent agreement with the analog computer results. The two others show little variation with α. It is to be noted that Raybaud, in a paper on the same question, points out the possibility of 5 roots. That certainly depends upon the aircraft and the calculations in question.

With numerical calculation of the coefficients of the equation in p, we found that many terms could be disregarded. If only the principal terms are retained, we are led to resolution of an equation of third degree whose roots can be expressed as

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$$p = \frac{V\alpha}{l} \frac{C_{l_\alpha} C_{n_j} - C_{n_\alpha} C_{l_j}}{C_{n_p} C_{l_j} - C_{n_j} C_{l_p} - \frac{(I_y - I_x)\,\rho\, C_{l_j}}{m\, l^2} \left[C_{z_\beta} - (C_{z_i} + C_x) \frac{C_{m_\beta}}{C_{m_i}} \right]}$$

$$p = \pm \frac{1}{2} \sqrt{\frac{-\rho V^2 S l\, C_{n_j} C_{m_i}}{(I_y - I_x) C_{m_i} + (I_z - I_x) C_{n_j}}}$$

In view of the simplifying hypotheses, it is interesting to compare the values of the root function of α with the values found with the computer and then examine the effect of the variation of certain parameters on a specific calculation. The case is M = 2.1, altitude 12 km (40,000 feet), 4° aileron deflection.

First, there is the effect of the yaw coupling due to roll

C_{n_p}	pº/s Computer	pº/s Theoretical
$2\ C_{n_{p_0}}$	150	144
$C_{n_{p_0}}$	105	112
0	85	88
$-\ 0.4\ C_{n_{p_0}}$	80	82

An excellent agreement is found, and it is seen that coefficient C_{n_p}, which is always poorly identified, has considerable influence. /5

The effect of the yaw coupling due to the ailerons producer

C_{n_α}	pº/s Computer	pº/s Theoretical
$C_{n_{\alpha_0}}$	90	55
0	85	88
$-\ C_{n_{\alpha_0}}$	70	82

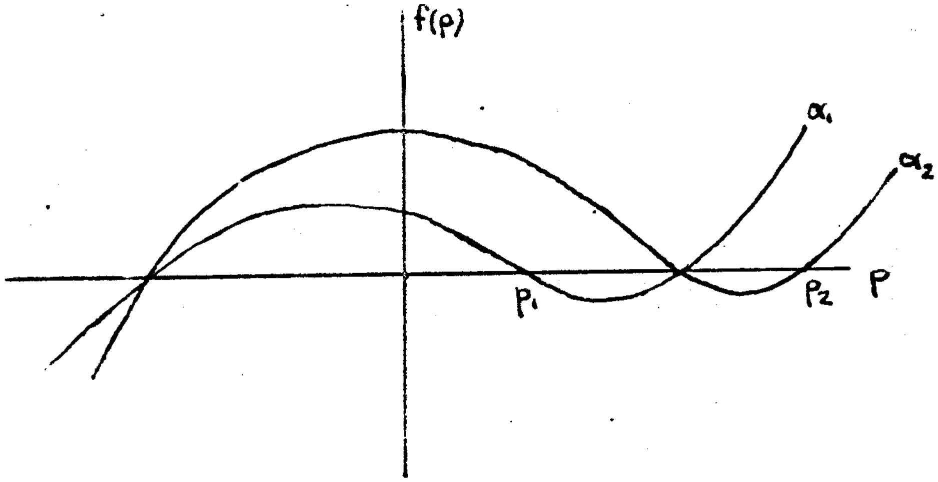
Finally we see the effect of elevator deflection

β	pº/s Computer	pº/s Theoretical
2º.8	150	149
- 4º.5	140	143

We find that the formula represents rather well the absolute values and variations of roll rate as a function of various parameters.

Let us now attempt to find the significance of the roots independent of α. We can effect all calculations by retaining the term in dp/dt. We find that $dp/dt = k\ f(p)/g(p)$, f(p) being the first member of the equation of the 11-th degree in p, and g(p) being a uniformly positive function because p = 0 up to a very high value. /6

We represent the variation of f(p) as function of p for two values of α.



If in the first case we are in the neighborhood of root p_1, an increase of the value of p yields $dp/dt < 0$ and movement is stable. In the second case, on the contrary, momement is unstable.

It can be noted moreover that in the case of stable movement, the value of p independent of α corresponds to unstable movement. This has already been observed by Professor Haus, using the analog computer, where depending upon the rapidity of the aileron deflection the standard stable solution is obtained, or an instable solution that corresponds to a clearly higher roll rate. /7

To ensure stability, assuming a slow aileron deflection, it is sufficient that the corresponding roll rate be less than the root independent of α. This is written:

$$\alpha < \frac{C_{n_p} C_{l_j} - C_{n_j} C_{l_p} - \frac{(I_y - I_x)\beta C_{l_j}}{m l^2} \cdot \left[C_{z_\beta} - (C_{z_i} + C_x)\frac{C_{m_\beta}}{C_{m_i}} \right]}{(C_{l_\alpha} C_{n_j} - C_{n_\alpha} C_{l_j})} \; \frac{1}{V} \sqrt{\frac{-\frac{q_0 \; S \; l \; C_{n_j}}{I_y - I_x}}{2\left[1 - \frac{I_x - I_z}{I_y - I_x} \; \frac{C_{n_j}}{C_{m_i}}\right]}}$$

Actually the effect of elevator reflection is weak and if it is neglected the equation is still further simplified.

It is surprising not to see a shift of the principal axis of inertia /8 with reference to the aircraft axis since it plays a major role in the coupling. This is due to the fact that we are working in the basis of the principal axes and the effect of inclination of the latter is expressed by aerodynamic coefficients that must be taken along this trihedron.

Let us now compare the limit values obtained by this criterion and those of the analog computer

Case studied		α limit (º)	α limit (º)
M	Z (km)	Computer	Theoretical
2,1	12	5.1	5.6
2,1	14	4.2	5
2,1	17	3.5	3.5
2,5	14	2	2.5
2.5	17	1.2	1.7
2.5	20	0.3	0.6

In view of the precision with which this limit can be determined by computer and the simplicity of the formula, it can be considered that the precision is excellent.

We have thus obtained a simple criterion of stability in the case of roll-pitch coupling. However, let us not forget that this study was related to a specific aircraft and that the admitted approximations run the risk of not being of value for any other craft. I would be personally very grateful to any persons who, having run into identical problems on the analog computer, would apply this criterion and see if it is general enough or not. /9

I close by thanking Miss Garnek for her extremely valuable assistance.